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COLLAPSE OF ONE-COMPONENT VAPOR BUBBLE WITH TRANSLATORY MOTION

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Abstract—Theoretical analysis of one-component vapor bubble collapse with translatory motion in uniformly subcooled liquid has been carried out. The bubble is spherical and flow in the region surrounding the bubble is potential. General solution is obtained in which the function $R = R(\tau)$ is defined implicitly by integral equation. General solution is reduced to the quasi steady state and quasi linear problem. Quasi steady state solution is used to obtain a set of simple and explicit expressions by which the bubble radius is determined in function of time. The results of theoretical analysis are compared with those given by other authors and available experimental data. The agreement between compared experimental data and theoretical results is very good.

NOMENCLATURE

a, thermal diffusivity $[m^2/s]$;
$A(\tau)$, function defined by equation (18) $[1/s]$;
Ar, $= gR_0^3/\nu^2$, Archimed number [dimensionless];
c_p, specific heat of liquid [J/kgK];
$E(H)$, function defined by equation (36) [dimensionless];
$F(H)$, function defined by equation (37) [dimensionless];
Fo, $= a\tau/R_0^2$, Fourier number [dimensionless];
g, gravitational acceleration constant $[m/s^2]$;
$G(\theta, \tau)$, function defined by equation (34) [dimensionless];
H, variable defined by equation (35) [dimensionless];
i, enthalpy of liquid [J/kg];
Δi, latent heat of evaporation [J/kg];
Ja, $= \dfrac{c_p \rho (T_i - T_\infty)}{\rho'' \Delta i}$, Jakob number [dimensionless];
K, coefficient defined by equation (53) [dimensionless];
K_σ, dimensionless group (50);
m, variable defined by equation (10) $[m^3]$;
$P(\theta, \tau)$, function defined by equation (16) $[s^{1/2}]$;
Pe_0, $= 2R_0 w_0/a$, Peclet number [dimensionless];
Pr, $= \nu/a$, Prandtl number [dimensionless];
r, radial position [m];
R, bubble radius [m];
R_0, initial bubble radius [m];
Re, $= 2R_0 w_0/\nu$, Reynolds number [dimensionless];
T, temperature [K];
w_0, translational velocity of bubble motion [m/s];
w_p, bulk liquid velocity (53) [m/s].

Greek symbols

β, $= R/R_0$, relative radius of bubble [dimensionless];
$\delta(\theta, \tau)$, thermal boundary layer thickness [m];
ε, dimensionless time (2);
ζ, resistance coefficient (51) [dimensionless];
θ, angle [rad];
λ, thermal conductivity of liquid [W/mK];
μ, dynamic viscosity of liquid $[Ns/m^2]$;
ν, kinematic viscosity of liquid $[m^2/s]$;
ρ, density of liquid $[kg/m^3]$;
σ, surface tension [N/m];
τ, time [s].

Subscripts

0, initial state;
∞, refers to value at great distance from the bubble;
i, refers to bubble surface.

Superscripts

$''$, refers to vapor phase.

INTRODUCTION

THE PROBLEM of the bubble collapse has been analyzed in several papers. Bankoff and Mikesell [1] are among the first who tried to make such an analysis. They analyzed the growth problem and bubble collapse considering only the influence of liquid inertia. Flourschuetz and Chao [2] have obtained solution investigating collapse mechanism directed by the heat-transfer process,

$$\varepsilon = \frac{2}{3}\frac{R_0}{R} + \frac{1}{3}\left(\frac{R}{R_0}\right)^2 - 1 \tag{1}$$

where dimensionless time is defined with

$$\varepsilon = \frac{4}{\pi} Ja^2 Fo. \tag{2}$$

Voloshko and Vurgaft [3] determined the bubble radius from macroscopic energy balance equation where for the form of temperature field is accepted solution of the problem of unsteady-state heat conduction through a semi-infinite body. For the bubble radius is obtained

$$\beta = 1 - 2\pi^{-1/2} Ja Fo^{1/2}. \tag{3}$$

In all these papers the collapse process has been analyzed as spherical symetric problem, in which the bubble center is motionless in relation to the liquid. It is certain however, that far greater technical importance has the collapse process of vapor bubbles which move relatively in relation to the liquid. There are a few theoretical papers on this problem. Wittke and Chao [4] have obtained numerical solutions for limiting cases of short and long time. Serious study of this problem was made by a group of authors whose results have been published in a series of papers [5–7]. General assumptions from [5, 6] have been used by Moalem and Sideman in [7] for direct quantitative analyzing of influence of the translatory motion velocity on the collapse process. Two solutions are shown;

$$\beta = [1 - \tfrac{3}{2}\pi^{-1/2} Ja Pe_0^{1/2} Fo]^{2/3} \tag{4}$$

for constant velocity of translatory motion (condition $w_0 = \text{const.}$)

$$\beta = [1 - \tfrac{5}{4}\pi^{-1/2} Ja Pe_0^{1/2} Fo]^{4/5} \tag{5}$$

for translatory motion velocity dependent on the bubble radius.

In [8] is presented the analytical solution of the problem of vapor bubble growth with translatory motion. In our paper general mathematical method for solution the basic differential equation from Ruckenstein's and Davis's paper [8] will be accepted (this method has also been analyzed in [9]). The difference between solution presented in this paper and the general solution of the bubble growth problem in [8] is based on the variety of differential equation forms which are the subject of analyzing [see equation (9) in [8] and (11) in this paper]. In that way we have obtained explicit solutions of the stagnant bubble collapse problem, as well as a set of simple relations from quasi steady state solution.

PROBLEM FORMULATION

The temperature field around the vapor bubble can be obtained from differential energy equation [8]

$$\frac{\partial T}{\partial \tau} + \left[\frac{R^2}{r^2}\frac{dR}{d\tau} - w_0\left(1 - \frac{R^3}{r^3}\right)\cos\theta\right]\frac{\partial T}{\partial \tau} + \frac{w_0}{r}\left(1 + \frac{R^3}{2r^3}\right)\sin\theta\frac{\partial T}{\partial \theta} = \frac{a}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) \tag{6}$$

and by satisfaction of boundary and initial conditions

$$T(r, \theta, 0) = T_\infty \tag{7}$$

$$T(\infty, \theta, \tau) = T_\infty \tag{8}$$

$$T(R, \theta, \tau) = T_i. \tag{9}$$

Introducing a new independent variable [10]

$$m = \tfrac{1}{3}[r^3 - R^3(\tau)] \tag{10}$$

and limiting to temperature change in a thin boundary layer near the bubble surface (where $3m/R \ll 1$) equation (6) can be approximated to

$$\frac{\partial T}{\partial \tau} - 3\frac{w_0}{R} m\cos\theta\frac{\partial T}{\partial m} + \frac{3}{2}\frac{w_0}{R}\sin\theta\frac{\partial T}{\partial \theta} = aR^4\frac{\partial^2 T}{\partial m^2}. \tag{11}$$

The boundary and initial conditions are

$$T(m, \theta, 0) = T_\infty \tag{12}$$

$$T(\infty, \theta, \tau) = T_\infty \tag{13}$$

$$T(0, \theta, \tau) = T_i. \tag{14}$$

GENERAL SOLUTION OF DIFFERENTIAL EQUATION

Differential equation (11) can be solved by mathematical method given in [8]. Because that its details will not be presented.

Together with boundary and initial conditions (12)–(14), the general solution of differential equation (11) is expressed with

$$T(m, \theta, \tau) = T_\infty + (T_\infty - T_i)\,\mathrm{erfc}\left[\frac{m}{\delta(\theta, \tau)}\right] \tag{15}$$

in which the thickness of temperature boundary layer is

$$\delta(\theta, \tau) = 2a^{1/2}\left\{\int_0^\tau R^4(p)\exp\left[\int^p \phi(s, \theta, \tau)\,ds\right]dp\right\}^{1/2} \equiv 2a^{1/2}P(\theta, \tau) \tag{16}$$

$$\phi(s, \theta, \tau) = 6A(s)\frac{1 - \tan^2(\theta/2)\exp\left[3\int^s A(e)\,de\right]}{1 + \tan^2(\theta/2)\exp\left[3\int^s A(e)\,de\right]} \tag{17}$$

$$A \equiv \frac{w_0(R)}{R}. \tag{18}$$

GENERAL SOLUTION FOR DETERMINING THE BUBBLE RADIUS

The bubble radius change in function of the temperature gradient on the interface of the bubble surface will be determined from the macroscopic energy balance equation written for the bubble as a whole,

$$\frac{\mathrm{d}R}{\mathrm{d}\tau} = -\frac{1}{2}\frac{\lambda}{\rho''\Delta i}\int_0^{\pi}\left(\frac{\partial T}{\partial r}\right)_{r=R}\sin\theta\,\mathrm{d}\theta. \tag{19}$$

Replacing (15)–(17) in (19) we get

$$R(\tau) = \frac{1}{1-\dfrac{R_0}{2}\left(\dfrac{a}{\pi}\right)^{1/2} Ja\displaystyle\int_0^{\tau}\int_0^{\pi}\frac{\sin\theta}{P(\theta,\xi)}\,\mathrm{d}\xi\,\mathrm{d}\theta}. \tag{20}$$

It is evident that the general solution for determining the bubble radius (20) is an implicit integral equation. For direct determining momentary values of the bubble radius should be used numerical technique. It is noteworthy that in the equation (20) the translatory motion velocity of the bubble centre is expressed by functional dependence

$$w_0 = f(R). \tag{21}$$

Therefore the general solution (20) cannot be used for explicit determination of $R(\tau)$ until the relation (21) is concrete.

Discussion on the further problem analysis will be connected with the solution of several special cases defined by certain specifications of the bubble translatory motion velocity (21).

Replacing (30) in (16) for the thickness of temperature boundary layer is obtained

$$\delta(\theta,\tau) = 2a^{1/2}[1+\tan^2(\theta/2)]^2\left\{\int_0^{\tau}\left|\frac{R(p)}{\exp[-\frac{3}{2}B(p-\tau)]+\tan^2(\theta/2)\exp[\frac{3}{2}B(p-\tau)]}\right|^4\mathrm{d}p\right\}^{1/2}$$
$$\equiv 2a^{1/2}P(\theta,\tau). \tag{31}$$

COLLAPSE OF A STAGNANT BUBBLE

One of the limiting cases of the analyzed problem is the bubble collapse process whose centre is motionless in relation to liquid.

From the problem definition it follows

$$w_0 = 0$$
$$A(\tau) = 0$$
$$\phi(s,\theta,\tau) = 0$$
$$P(\tau) = \left|\int_0^{\tau} R^4(p)\,\mathrm{d}p\right|^{1/2} \tag{22}$$

Using (22) it is obtained from (20)

$$\frac{R_0}{R(\tau)} = 1 - R_0 Ja\left(\frac{a}{\pi}\right)^{1/2}\int_0^{\tau}\frac{\mathrm{d}s}{\left|\displaystyle\int_0^{s}R^4(p)\,\mathrm{d}p\right|^{1/2}}. \tag{23}$$

In differential form the equation (23) is as follows

$$S = R_0 Ja\left(\frac{a}{\pi}\right)^{1/2}\left|\int_0^{\tau}R^4(p)\,\mathrm{d}p\right|^{-1/2} \tag{24}$$

where the used sign was

$$S \equiv \left(\frac{\mathrm{d}R}{\mathrm{d}\tau}\frac{R_0}{R^2}\right). \tag{25}$$

With new differentiation, using (24), (25), we get

$$\frac{\mathrm{d}S}{\mathrm{d}R} = \frac{\pi}{2aR_0Ja^2}(SR)^2. \tag{26}$$

It follows from (26) that,

$$R^3 - \left|3R_0^2 + \frac{12aJa^2}{\pi}\tau\right|R + 2R_0^3 = 0. \tag{27}$$

From the solution of the cubic equation (27) we find

$$\beta(\tau) = 2(1+\varepsilon)^{1/2}$$
$$\times\cos\{\tfrac{1}{3}[\pi + \arccos(1+\varepsilon)^{-3/2}]\}. \tag{28}$$

The obtained solution (28) is in qualitative agreement with the solution of the bubble growth problem shown in [10].

COLLAPSE OF VAPOR BUBBLE DEFINED AS QUASI LINEAR PROBLEM

The assumption of quasi linearity of the problem brings to a statement that the velocity of the bubble translatory motion and its radius are connected with linear dependence

$$w_0 = BR. \tag{29}$$

The condition (29) leads to the approximation

$$A(\tau) = B = \text{const.} \tag{30}$$

For determining the bubble radius in function of time still general solution (20) is valid, in which assuming the quasi linearity of the problem, the function $P(\theta,\tau)$ is specified through (31).

COLLAPSE OF VAPOR BUBBLE DEFINED AS QUASI STEADY STATE PROBLEM

The assumption of quasi steady state problem leads to the assertion that the bubble radius change during the collapse has not any essential influence on the form of the temperature field, and according to that neither the thickness of the boundary layer.

The mentioned approximation is analytically expressed as the assumption

$$A, R = \text{const.} \tag{32}$$

in expressions by which the thickness of the boundary layer is determined.

Assuming the condition (32), equation (16) becomes

$$\delta(\theta,\tau) = \frac{4R^2}{3\sin^2\theta}\left(\frac{2a}{A}\right)^{1/2}G(\theta,\tau) \tag{33}$$

$$G(\theta,\tau) = \left\{\frac{1+3\tan^2(\theta/2)\exp(-H)}{[1+\tan^2(\theta/2)\exp(-H)]^3} - \frac{1+3\tan^2(\theta/2)}{[1+\tan^2(\theta/2)]^3}\right\}^{1/2} \quad (34)$$

in which dimensionless time is designated with H

$$H \equiv 3A\tau. \quad (35)$$

The function (34) is qualitatively identical with solution obtained in [11].

Bubble radius can be determined from the equation which is obtained by replacing (33) in the general solution (20)

$$[A(R)]^{-1/2}\,\mathrm{d}R = \frac{3}{8}Ja\left(\frac{2a}{\pi}\right)^{1/2} \times \left|\int_0^{\pi}\frac{\sin^3\theta}{G(\theta,\tau)}\,\mathrm{d}\theta\right|\mathrm{d}\tau \equiv \frac{Ja}{8A}\left(\frac{2a}{\pi}\right)^{1/2} E(H)\,\mathrm{d}H \quad (36)$$

$$\int_{R_0}^{R}[A(R)]^{-1/2}\,\mathrm{d}R = \frac{Ja}{8A}\left(\frac{2a}{\pi}\right)^{1/2} \times \int_0^H E(s)\,\mathrm{d}s \equiv \frac{Ja}{8A}\left(\frac{2a}{\pi}\right)^{1/2} F(H). \quad (37)$$

It is noteworthy that by increasing the value H the function $E(H)$ converges very quickly to

$$\lim_{H\to\infty} E(H) = \int_0^{\pi}\frac{\sin^3\theta\,\mathrm{d}\theta}{\left\{1 - \dfrac{1+3\tan^2(\theta/2)}{[1+\tan^2(\theta/2)]^3}\right\}^{1/2}} = \frac{8}{3}. \quad (38)$$

Numerical analysis shows that convergence of the given integral is practically provided by satisfying the condition

$$|H| \geqq 3. \quad (39)$$

Under such circumstances when validity of condition (39) is provided the bubble radius is determined by very simple dependence

$$\int_{R_0}^{R}[A(R)]^{-1/2}\,\mathrm{d}R = Ja\left(\frac{2a}{\pi}\right)^{1/2}\tau. \quad (40)$$

EXAMPLES OF EXPLICIT EXPRESSIONS FOR DETERMINING THE BUBBLE RADIUS

Our basic intention is to illustrate the method and to get the last forms of solution in which the bubble radius can be defined by functional expressions of explicit type convenient for wide technical use. For that purpose it should perform integration (37) or (40) with adopted dependences of the bubble translatory motion velocity (21). From practical reasons we will accept validity of condition (39) where the bubble radius is defined from the simplest dependence (40).

It is evident that any function (21), integral on R, can be included in (40). It gives great possibilities for relatively large numbers of existing expressions used for determining thermal velocity of vapor or gas bubble rise to be included in (40) and in that way a set of different expressions for determining $R = R(\tau)$ can be obtained. We shall limit ourselves to illustrations of this method by accepting several well known forms (21).

We shall accept five characteristic cases of different gas bubble flow regimes.

Case I: *Constant translational velocity*

Under the condition

$$w_0 = \text{const.}$$

it follows from (40) that

$$\beta = [1 - \tfrac{3}{2}Ja\pi^{-1/2}Pe_0 Fo]^{2/3}. \quad (41)$$

The solution (41) is the same as obtained in [7] [see equation (4)].

Case II: *Radius dependent translational velocity in Stokes flow* (Re < 2)

In Stokes flow regime ($Re < 2$) translational velocity of the bubble motion is defined as

$$w_0 = \frac{2}{9}\frac{g}{\nu}H_d R^2 \quad (42)$$

where H_d is the Hadamard factor defined by [12]

$$H_d = 1$$

(without internal circulation)

$$H_d = \left|\frac{1+(\mu''/\mu)}{(2/3)+(\mu''/\mu)}\right|$$

(with internal circulation).

In this case we find from (40) that

$$\beta = [1 - \tfrac{1}{3}(H_d/\pi)^{1/2}(ArPr)^{1/2}JaFo]^2. \quad (43)$$

Case III: *Radius dependent translational velocity* (Re > 2)

In this case the bubble flow is characterised by appearance of internal circulation, but the bubble's shape is still spherical.

The upper boundary of this regime is defined by [13]

$$Re = 4.02\left(\frac{\rho\sigma^3}{g\mu^4}\right)^{0.214}. \quad (44)$$

The translational rise velocity is defined by empirical expression [12]

$$w_0 = 0.347 g^{3/4}\nu^{-1/2}R^{5/4}. \quad (45)$$

Substituting (45) into equation (40) we obtain

$$\beta = [1 - \tfrac{7}{8}(0.694/\pi)^{1/2}Ar^{3/8}Pr^{1/2}JaFo]^{8/7}. \quad (46)$$

Case IV: *Radius dependent translational velocity with bubbles of variable shape*

In this regime of flow the bubble's shape is variable (it is mostly approximated by oblate spheroid).

The lower boundary of this case is defined by the Reynolds number calculated from (44).

The upper boundary is defined by [13]

$$Re = 3.1\left(\frac{\rho\sigma^3}{g\mu^4}\right)^{0.25} \tag{47}$$

The translational rise velocity can be obtained from [12–14]

$$w_0 = 1.35(\sigma/\rho)^{1/2}R^{-1/2}. \tag{48}$$

Using (48), from (40) we have

$$\beta = [1 - \tfrac{7}{4}(2.7/\pi)^{1/2}K_\sigma^{1/4}JaFo]^{4/7} \tag{49}$$

where

$$K_\sigma = \left(\frac{R_0\sigma}{\rho a^2}\right). \tag{50}$$

Case V: *Radius dependent translational velocity* (Re > 20)

In this case, we shall accept that the translational rise velocity can be defined by [15]

$$w_0 = \left(\frac{8}{3}\frac{g}{\zeta}R\right)^{1/2} \tag{51}$$

where the resistance coefficient ζ is constant.

It follows from (51) and (40) that

$$\beta = [1 - \tfrac{5}{2}(6\zeta\pi^2)^{-1/4}Ar^{1/4}Pr^{1/2}JaFo]^{4/5}. \tag{52}$$

DISCUSSION AND COMPARISON WITH EXPERIMENTAL DATA

It can be seen that the general solution which defines the bubble radius change during condensation is obtained as integral equation (20). It can be solved only numerically. Numerical integration method requests the assumption of the bubble's translatory motion velocity w_0. While there are no actual basis for iteration numerical procedure on w_0, the general solution can be directly analyzed from the point of influence which has the absolute translational velocity on the character of the bubble radius change. This sort of qualitative analysis is, for the bubble growth process, presented in [8]. Because the results of such a numerical analysis of the general solution (20) are qualitatively identical with [8] (with change of the sign in the bubble growth equation, while the condensation process is in question, see discussion in [16]) they will not be presented in this paper.

In contrast to the general solution, by assuming the quasi linearity of the problem, the result (31) is obtained for direct use of which need not accept the velocity w_0. Translational velocity is according to (42), (45), (48), (51) actually proportional with the bubble radius. The assumption (30) actually means the linear approximation of this dependence. It should be pointed out that utilizing of quasi linear solution makes the numerical iteration technique much easier.

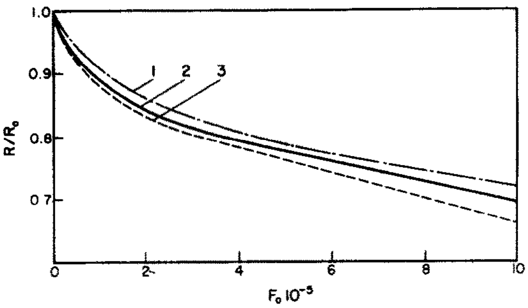

Fig. 1. Comparison between theoretical solutions for collapse of stagnant bubble. 1. Equation (1), [2]; 2. Equation (28), this paper; 3. Equation (3), [3].

The assumption of quasi steady state problem enables solutions by which the bubble radius is defined by functional dependences of explicit type. Because of evident convenience of those solutions in relation to the wide technical application we shall give a special attention to the possibilities of their use.

First we shall compare equation (28), which is valid for collapse of stagnant bubbles, with the results given by the other authors (1) [2] and (3) [3]. The results are presented in Fig. 1.

Curves in Fig. 1 are drawn for the case if the value $Ja = 30$. It can be seen that all the three present solutions give the curves of the same character. The differences between absolute values are within the range of $\pm 6\%$.

For the direct practical using of quasi steady state solution (33) the functions $E(H)$ and $F(H)$ should be determined. Some characteristic values obtained by numerical determinations of functions $E(H)$ (36) and $F(H)$ (37) are given in the Table 1. The accepted initial value of function $F(H)$ is given with $F(H) = 0$ when $H = 0.001$.

The earlier pointed fact connected with convergence of the function $E(H)$ is evident. The condition (39) is not fulfilled only in the initial period of the condensation process. Time during which the condition (39) will be fulfilled depends on absolute value R_0 and thermophysical properties. To illustrate the absolute value of lasting that part of the process, let us give a

Table 1

$H =$	0.001	0.005	0.01	0.05	0.15
$E(H) =$	103.168	46.1828	32.6552	14.608	8.44$\dot{6}$
$F(H) =$	0	0.2987	0.49578	1.441	2.5938
$H =$	0.2	0.3	0.5	1	2
$E(H) =$	7.323	5.9998	4.69792	3.48186	2.84294
$F(H) =$	2.988	3.6542	4.7239	6.76884	9.93124
$H =$	3	5	10	100	
$E(H) =$	2.7052	2.6676	2.6666	2.6666	
$F(H) =$	12.7054	18.078	31.416	271.416	

small numerical example with real accepted values for w_0 and R_0. Let it be: $w_0 = 30 \cdot 10^{-2}$ [m], $R_0 = 0.57 \cdot 10^{-3}$ [m]. Under given conditions the time when the condition (39) is fulfilled is $\tau = 1.9 \cdot 10^{-3}$ [s].

In the further discussion we shall assume that the condition (39) is fulfilled during the whole condensation process.

It has already been pointed out to the fact that solutions (4) [7] and (41) are identical. On the other hand solutions (5) [7] and (52) are of the same character.

Voloshko and Vurgaft [3, 17] have published experimental results on the vapor bubble collapse in distilled water. The initial bubble radius was in the range of $R_0 = (5\text{–}12.5)10^{-3}$ [m], while the water was subcooled to $\Delta T = (5\text{–}25)$[K]. The process was performed by gravitational rising of bubbles through macroscopic stagnant liquid. Unfortunately in [17] the presented experimental results are given without precise definition R_0, w_0 and Ja for each of experimental points. Instead of that empirical correlation for determining initial bubble radius was given [3]

$$R_0 = 0.0295\Delta T^{-0.53}\,[\text{m}].$$

All the experimental results on the bubble radius change have been correlated by a solid line in Fig. 2 [3]. According to [3] the deviation in correlating of experimental results was within the range of $\pm 30\%$.

From Fig. 2, it can be seen that the experimental correlative curve and theoretical solution (46) are in qualitative agreement. Deviation of theoretical solution from experimental correlative curve is less than $\pm 30\%$.

Abdelmessih, Hooper and Nanngia [18] have reported experimental results on the effect of forced liquid flow on the bubble growth and bubble collapse in distilled water. Direction of bulk liquid flow was vertical on the direction of the bubble rise motion. After detachment from the heating surface the bubble was exposed to the effect of horizontal component of the bulk liquid flow w_p and vertical component of the bubble gravitational rise velocity w_0. The resulting velocity is

$$w = w_0[1+(w_p/w_0)^2]^{1/2} = w_0(1+K)^{1/2}. \quad (53)$$

In order to be able to compare our theoretical results with the experiments we have calculated the factor K on the basis of experimental values for w_p (from [18]) and values for w_0 from (42) (because the bubbles in experiments are very small, the largest has dimension $R_{0\max} = 0.256 \cdot 10^{-3}$ [m]). The bubble radius has been figured out on the basis of solution (46) because the resulting flow takes place with $Re > 2$. Flow velocity has been calculated on the basis of equation (53). Comparison of the results is given in Fig. 3.

From Fig. 3 it can be noticed that the agreement between theoretical solution (46) and experimental results is both qualitatively and quantitatively good. It is also pointed out to an expected phenomena that by increasing the bulk liquid flow velocity the collapse rate is also increased.

On this occasion we can point out to the characteristic way of using the solution (46) in the collapse of the bubble with the forced bulk liquid flow by introducing relation (53). The presented analysis of the given example points out to the possibility of application of solutions obtained for collapse with gravitational bubble rise motion on the complicated collapse problems with forced liquid flow. It is evident that in the case of the forced flow the collapse rate could be

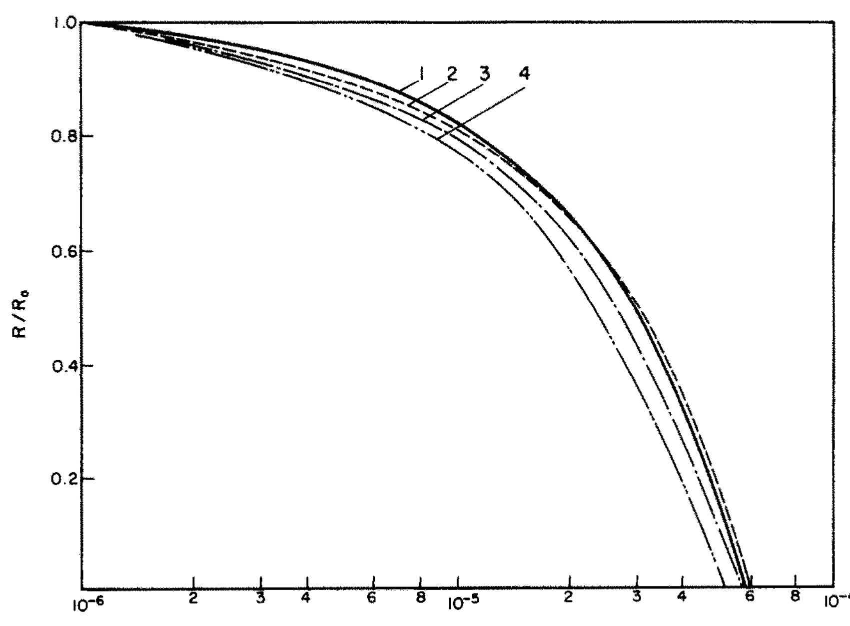

FIG. 2. Comparison between theory and experiments [3]. 1. Experiments [3]; 2. Equation (46), $Ja = 30$; 3. Equation (46), $Ja = 45$; 4. Equation (46), $Ja = 60$.

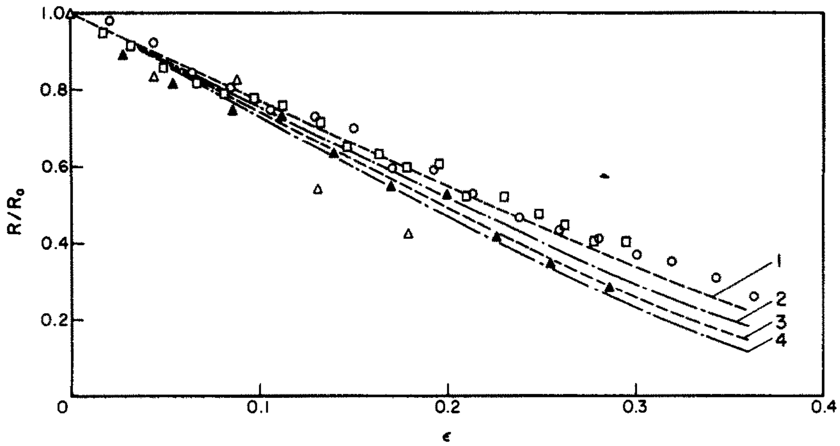

FIG. 3. Comparison between theory and experiments [18]. 1. Equation (46), $R_0 = 0.256 \cdot 10^{-3}$ m; $K = 5.4$. 2. Equation (46), $R_0 = 0.218 \cdot 10^{-3}$ m; $K = 14.77$. 3. Equation (46), $R_0 = 0.184 \cdot 10^{-3}$ m; $K = 39.65$. 4. Equation (46), $R_0 = 0.16 \cdot 10^{-3}$ m; $K = 90.55$. □, [18]. $R_0 = 0.256 \cdot 10^{-3}$ m; $w_p = 1.142$ m/s. ○, [18], $R_0 = 0.218 \cdot 10^{-3}$ m; $w_p = 1.37$ m/s. ▲, [18], $R_0 = 0.184 \cdot 10^{-3}$ m; $w_p = 1.599$ m/s. △, [18], $R_0 = 0.16 \cdot 10^{-3}$ m; $w_p = 1.827$ m/s.

determined from adequate solution of the collapse problem with the gravitational bubble rise velocity in which it should be included the resultant flow velocity [defined by the relationship of the type (53)]. For generalization of this assumption it should possess more experimental results.

CONCLUSIONS

1. Collapse of single one-component vapor bubble is defined by the general implicit solution (20), the solutions obtained by assuming the quasi linearity of the problem (31) and the solutions obtained by the assuming quasi steady state problem (37), (40).

2. Analysis of the quasi steady state solution (40) is performed under the assumption that the actual translational velocity during the collapse of the bubble can be approximated with thermal velocity of the bubble rise motion. In that way for different flow regimes a set of explicit functional dependences was obtained for determining of collapse rate.

3. Expression (28), obtained from the general solution (20) as the special case valid for the collapse of stagnant bubble is compared with those by other authors. The agreement between the results is within the deviation range of $\pm 6\%$.

4. Explicit solutions based on the quasi steady state model are compared with available experimental data [3, 17, 18]. A good agreement is shown, especially with experiments [18]. It was indicated to the possibility of determining the collapse rate with forced bulk liquid flow by using a simple method of introducing the resultant flow velocity in the expressions obtained as the solutions of the quasi steady state collapse with the gravitational bubble rise motion.

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COLLAPSUS D'UNE BULLE DE VAPEUR A UN SEUL COMPOSANT AVEC MOUVEMENT DE TRANSLATION

Résumé—On analyse théoriquement le collapsus d'une bulle de vapeur avec un mouvement translatoire dans un liquide uniformément sous-refroidi. La bulle est sphérique et l'écoulement dans la région proche de la bulle est potentiel. On obtient une solution générale dans laquelle la fonction $R = R(\tau)$ est définie implicitement par une équation intégrale. La solution générale est réduite à celle d'un problème quasi linéaire d'état quasi permanent. La solution de l'état quasi permanent est utilisée pour obtenir un système d'expressions simples et explicites par lesquelles le rayon de bulle est déterminé en fonction du temps. Les résultats de l'analyse théorique sont comparés avec ceux donnés par d'autres auteurs et avec des résultats expérimentaux. Dans la comparaison entre données expérimentales et résultats théoriques, l'accord est très bon.

DAS ZUSAMMENBRECHEN EINER TRANSLATORISCH BEWEGTEN EINKOMPONENTEN-DAMPFBLASE

Zusammenfassung—Das Zusammenbrechen einer in einer gleichförmig unterkühlten Flüssigkeit translatorischbewegten Einkomponenten-Dampfblase wurde theoretisch untersucht. In der Umgebung der kugelförmigen Blase wurde Potentialströmung vorausgesetzt. Es wurde eine allgemeine Lösung gefunden, wobei die Funktion $R = R(\tau)$ implizit durch eine Integralgleichung gegeben ist. Aus der allgemeinen Lösung ergeben sich die Lösungen für den quasistationären und den quasilinearen Fall. Aus der Lösung für den quasistationären Fall werden einfache explizite Ausdrücke für den Blasenradius in Abhängigkeit von der Zeit abgeleitet. Die Ergebnisse der theoretischen Untersuchung werden mit denen anderer Autoren und mit Meßwerten verglichen. Die Übereinstimmung zwischen experimentellen und theoretischen Ergebnissen ist sehr gut.

РАЗРУШЕНИЕ ОДНОКОМПОНЕНТНОГО ПАРОВОГО ПУЗЫРЯ ПРИ ПОСТУПАТЕЛЬНОМ ДВИЖЕНИИ

Аннотация — Проводится теоретическое исследование разрушения однокомпонентного парового пузыря при поступательном движении в равномерно охлажденной жидкости. Пузырь имеет сферическую форму, течение вокруг пузыря носит потенциальный характер. Получено общее решение, в котором функция $R = k(\tau)$ определена с помощью интегрального уравнения в неявной форме. Общее решение сводится к квазистационарной и квазилинейной задаче. Квазистационарное решение используется для получения системы простых и явных выражений, на основе которых определяется радиус пузыря в функции времени. Результаты теоретического анализа сравниваются с результатами других авторов и с имеющимися экспериментальными данными. Получено довольно хорошее соответствие между экспериментальными данными и теоретическими результатами.